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ON THE GROUP OF HOMEOMORPHISMS OF THE BASILICA

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Abstract. We show that the group of all homeomorphisms of the Basilica fractal coincides with a group of transformations of a certain non-locally finite ribbon tree (a ribbon graph is a graph with a fixed cyclic order of edges at each vertex). Also, we show that the Basilica Thompson group defined by James Belk and Bradley Forrest is dense in the group of all orientation preserving homeomorphisms of the Basilica.

1. The Basilica

The Basilica $\mathcal{B}$ is a compact topological space obtained as a quotient of the circle $S = \mathbb{R}/2\mathbb{Z}$ under the equivalence relation

$$\frac{3k+1}{3^n} \sim \frac{3k+2}{3^n}, \text{where } k \text{ and } n \text{ are nonnegative integers.} \tag{1.1}$$

In particular, $1 \sim 2 = 0$. In other words, we write numbers in **a** ternary system and identify $a_0.a_1 \dots a_{n-1}1 \sim a_0.a_1 \dots a_{n-1}2$. The reader may find nice pictures of the Basilica in [7], [1], and [2] or their preprint versions in `arXiv`; see also Figure 1.

We say that a *separating point* of $\mathcal{B}$ is a point corresponding to a nontrivial (two-point) equivalence class. Denote by $\mathrm{Sep}(\mathcal{B})$ the set of separating points. If $x \in \mathrm{Sep}(\mathcal{B})$, then $\mathcal{B}\backslash x$ consists of two components. Otherwise, $\mathcal{B}\backslash x$ is connected.

It is convenient to think that our circle S is a boundary of a disk D, and that equivalent points are connected by chords $[a—b]$ (*leafs*); clearly,

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