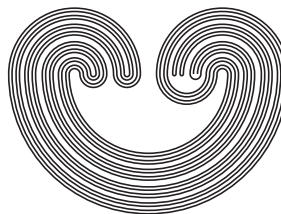


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## SURVEY ON INVARIANT QUASIMORPHISMS AND STABLE MIXED COMMUTATOR LENGTH

by

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## **SURVEY ON INVARIANT QUASIMORPHISMS AND STABLE MIXED COMMUTATOR LENGTH**

MORIMICHI KAWASAKI, MITSUAKI KIMURA, SHUHEI MARUYAMA,  
TAKAHIRO MATSUSHITA, AND MASATO MIMURA

**ABSTRACT.** A homogeneous quasimorphism  $\phi$  on a normal subgroup  $N$  of  $G$  is said to be  $G$ -invariant if  $\phi(gxg^{-1}) = \phi(x)$  for every  $g \in G$  and for every  $x \in N$ . Invariant quasimorphisms have naturally appeared in symplectic geometry and the extension problem of quasimorphisms. Moreover, it is known that the existence of non-extendable invariant quasimorphisms is closely related to the behavior of the stable mixed commutator length  $\text{scl}_{G,N}$ , which is a certain generalization of the stable commutator length  $\text{scl}_G$ .

In this survey, we review the history and recent developments of invariant quasimorphisms and stable mixed commutator length. The topics we treat include several examples of invariant quasimorphisms, Bavard's duality theorem for invariant quasimorphisms, Aut-invariant quasimorphisms, and the estimation of the dimension of spaces of non-extendable quasimorphisms. We also mention the extension problem of partial quasimorphisms.

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*Key words and phrases.* invariant quasimorphisms, quasimorphisms, stable commutator lengths, stable mixed commutator lengths.

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