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by

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THE BOUNDARY RIGIDITY OF LATTICES IN PRODUCTS OF TREES

KASIA JANKIEWICZ, ANNETTE KARRER, KIM RUANE, AND BAKUL SATHAYE

Abstract. We show that every group acting properly and cocompactly by isometries on a product of n bounded valance, bushy trees is boundary rigid. That means that every CAT(0) space that admits a geometric action of any such group has the visual boundary homeomorphic to a join of n copies of the Cantor set.

1. Introduction

A visual boundary is a particular type of compactification of a proper CAT(0) metric space. The boundary is defined as a set of equivalence classes of asymptotic rays endowed with an appropriate topology. For hyperbolic spaces X and Y, any quasi-isometry $X \to Y$ between them extends to a homeomorphism of their visual boundaries. Consequently, the homeomorphism type of the boundary of a hyperbolic group is a well-defined group invariant. This is not true for CAT(0) groups, i.e., groups that act geometrically on CAT(0) spaces.

Bowers and Ruane [BR96] give an example of a group G acting geometrically on CAT(0) spaces X and Y, such that the associated G-equivariant quasi-isometry between the spaces does not extend to a homeomorphism

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