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SEQUENCES OF COMPLEX RADIUS VALUES THAT YIELD CAPTURE SIERPIŃSKI CURVE JULIA SETS FOR n-CIRCLE INVERSION

DANIEL M. LOOK

Abstract. The rational maps $z \mapsto \frac{r^2 z^{n-1}}{z^n - 1}$ are related to the geometric action of circle inversion. We prove that for n odd, there exist multiple sequences of radii in parameter space that yield Sierpiński curve Julia sets. Further, although any two such (distinct) radii will yield homeomorphic Julia sets, the dynamics of the functions restricted to their Julia sets are not conjugate.

1. Introduction

The map sending a point to its image under circle inversion is dynamically uninteresting as every point is either fixed or lies on a two-cycle. However, if we define a system via multiple circle inversions the dynamics become more complex. Specifically, we will map a point z to the arithmetic mean of the n inversions of z about circles centered at the n^{th} roots of unity with common radius r. This yields the map $z \mapsto \frac{r^2 \bar{z}^{n-1}}{\bar{z}^n - 1}$. We will use the related holomorphic map

$$F_{r,n}(z) = \frac{r^2 z^{n-1}}{z^n - 1},$$

which we will refer to as *n-circle inversion.* The holomorphic and anti-holomorphic maps have been studied previously; see, for example, [4], [5], and [6].

In [5] and [6], the circles are allowed to have complex radii; a circle with radius $r = |r|e^{i\theta_r}$ can be thought of as the image of $\theta \mapsto re^{i\theta} = |r|e^{i(\theta+\theta_r)}$.

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