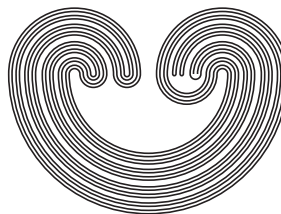


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## VIETORIS ENDOFUNCTOR FOR CLOSED RELATIONS AND ITS DE VRIES DUAL

by

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## VIETORIS ENDOFUNCTOR FOR CLOSED RELATIONS AND ITS DE VRIES DUAL

MARCO ABBADINI, GURAM BEZHANISHVILI, AND LUCA CARAI

**ABSTRACT.** We generalize the classic Vietoris endofunctor to the category of compact Hausdorff spaces and closed relations. The lift of a closed relation is done by generalizing the construction of the Egli–Milner order. We describe the dual endofunctor on the category of de Vries algebras and subordinations. This is done in several steps, first by generalizing the known endofunctor on the category of boolean algebras and boolean homomorphisms, then lifting it up to  $S5$ -subordination algebras, and finally using MacNeille completions to further lift it to de Vries algebras. Among other things, this yields a generalization of Johnstone’s pointfree construction of the Vietoris endofunctor to the category of compact regular frames and preframe homomorphisms.

### 1. INTRODUCTION

Hyperspace constructions are among classic constructions in topology [21] with numerous applications [23]. One of the most famous is taking the Vietoris space  $\mathbb{V}(X)$  of a compact Hausdorff space  $X$ . It is well known

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*Key words and phrases.* closed relation, compact Hausdorff space, compact regular frame, de Vries algebra, Gleason cover, ideal completion, MacNeille completion, proximity, subordination relation, Vietoris space.

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