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THE CONNECTEDNESS OF SUBSETS IN A CONTINUUM IMPLIES CONNECTEDNESS OF VIETORIC SETS IN THE HYPERSPACE $C_n(X)$

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Abstract. Let X be a continuum and n be a positive integer. The symbol $C_n(X)$ denotes the hyperspace of all nonempty, closed subsets of X having at most n components, equipped with the Vietoris topology. Given a finite family of subcontinua of X, $\{C_1, \ldots, C_r\}$, it is well known that the set $\langle C_1, \ldots, C_r\rangle_n$ defined as the set of all elements A in $C_n(X)$ such that $A \subset \bigcup_{i=1}^{r} C_i$ and $A \cap C_i \neq \emptyset$ for each i, is a subcontinuum of $C_n(X)$. In this paper, we extend the previous result by showing that, if each C_i is connected (arcwise connected) and $r \leq n$, then $\langle C_1, \ldots, C_r\rangle \cap C_n(X)$ is a connected (arcwise connected) subspace of $C_n(X)$.

1. Introduction

A *continuum* is a nondegenerate, compact, connected metric space. Given a continuum X, a *subcontinuum* of X is a subspace of X which is nonempty, closed, and connected. Given a positive integer n, we consider the following hyperspaces of X:

- $2^X = \{A \subset X \ : \ A \text{ is nonempty and closed in } X\}$,
- $F_n(X) = \{A \in 2^X \ : \ A \text{ has at most } n \text{ points}\}$,
- $C_n(X) = \{A \in 2^X \ : \ A \text{ has at most } n \text{ components}\}$.

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