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S. K. Maity and Gopal Adak

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Web:	http://topology.nipissingu.ca/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
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TOPOLOGICAL SEMIRINGS WHICH ARE UNION OF TOPOLOGICAL RINGS

S. K. MAITY AND GOPAL ADAK

Abstract. In this paper, we establish some necessary and sufficient conditions on a topological completely regular (algebraic) semiring to be a union of topological skew-rings. As a consequence we show that a topological completely regular (algebraic) semiring (S, τ) satisfying the identity $x + 0_y + 0_x + y = 0_x + y + x + 0_y$ is an idempotent semiring of topological rings if and only if for each $G \in \tau$ and every $a \in G$, there exists an element $U \in \tau$ such that $a \in U \subseteq G \cap H_a^+$. Also, we prove that the quotient space S/K of a topological completely regular (algebraic) semiring (S, τ) by a normal ideal K is a topological idempotent semiring. Finally, we establish that the quotient space of topological product of a family of topological semirings by product of their normal ideals is topologically isomorphic with the topological product of the family of quotient spaces of topological semiring by its corresponding normal ideal.

1. Introduction

A semiring $(S, +, \cdot)$ is a non-empty set S together with two binary operations '$+$' and '$\cdot$' on S such that semigroup reducts $(S, +)$ and $(S, \cdot)$ are connected by distributive laws, i.e., $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ for all $a, b, c \in S$. In addition, if the additive reduct $(S, +)$ of a semiring S is a group, then the semiring S is said to be a skew-ring. Thus, a skew-ring S is a ring if and only if addition is commutative.

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