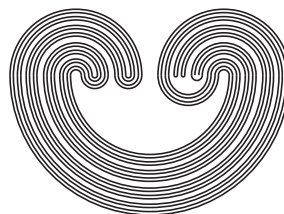

TOPOLOGY PROCEEDINGS

Volume 65, 2025

Pages 75–97



ANOTHER TYPE OF QUOTIENTS OF HYPERSPACES

by

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Electronically published on November 7, 2024

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Auburn University, Alabama 36849, USA

E-mail: topolog@auburn.edu

ISSN: (Online) 2331-1290, (Print) 0146-4124

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ANOTHER TYPE OF QUOTIENTS OF HYPERSPACES

FERNANDO GARCÍA AND SERGIO MACÍAS

In Memoriam Professor Piotr Minc

ABSTRACT. Let X be a continuum, and let n be a positive integer. We introduce a new type of quotient of hyperspaces. For this, consider the hyperspace 2^X of X , consisting of all nonempty closed subsets of X , and the n -fold hyperspace of X , $\mathcal{C}_n(X)$, whose elements are all nonempty closed subsets of X with at most n components. These hyperspaces are topologized with the Hausdorff metric. We define the quotient space $2_n^X = 2^X / \mathcal{C}_n(X)$, with the quotient topology. We call 2_n^X the n -fold suspension hyperspace of X . Note that this is the first time that a quotient of the hyperspace 2^X has been taken. We prove several properties of n -fold suspension hyperspaces. For example: we show that 2_n^X is a unicoherent continuum. We give sufficient conditions to have that 2_n^X is contractible. We prove that the continuum X is locally connected if and only if 2_n^X is the Hilbert cube. Given a map $f: X \rightarrow Y$ between continua, we define and study the corresponding induced map between the n -fold suspension hyperspaces of X and Y .

1. INTRODUCTION

A *continuum* is a compact, connected, metric space. Given a continuum X , we consider its hyperspaces: 2^X , consisting of the family of nonempty closed subsets of X ; $\mathcal{C}_n(X)$, whose elements belong to 2^X and each has

2020 *Mathematics Subject Classification.* 54B20, 54F15, 54F16.

Key words and phrases. Absolute retract, continuum, decomposable continuum, dendroid, fan, hereditarily decomposable continuum, hereditarily indecomposable continuum, hereditarily unicoherent continuum, hyperspaces, indecomposable continuum, induced map, n -fold suspension hyperspace, property of Kelley, smooth fan, trivial shape, unicoherent continuum.

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