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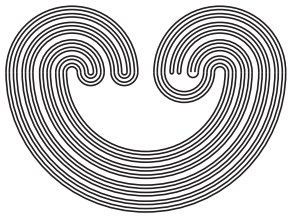

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Another type of quotients of hyperspaces

by

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ANOTHER TYPE OF QUOTIENTS OF HYPERSPACES

FERNANDO GARCÍA AND SERGIO MACÍAS

In Memoriam Professor Piotr Minc

Abstract. Let X be a continuum and let n be a positive integer. We introduce a new type of quotient of hyperspaces. For this, consider the hyperspace 2^X of X, consisting of all nonempty closed subsets of X, and the n-fold hyperspace of X, $\mathcal{C}_n(X)$, whose elements are all nonempty closed subsets of X with at most n components. These hyperspaces are topologized with the Hausdorff metric. We define the quotient space $2^X_n = 2^X/\mathcal{C}_n(X)$, with the quotient topology. We call 2^X_n the *n-fold suspension hyperspace of X*. Note that this is the first time that a quotient of the hyperspace 2^X has been taken. We prove several properties of n-fold suspension hyperspaces. For example: we show that 2^X_n is a unicoherent continuum. We give sufficient conditions to have that 2^X_n is contractible. We prove that the continuum X is locally connected if and only if 2^X_n is the Hilbert cube. Given a map $f\colon X \to Y$ between continua, we define and study the corresponding induced map between the n-fold suspension hyperspaces of X and Y.

1. Introduction

A *continuum* is a compact, connected, metric space. Given a continuum X, we consider its hyperspaces: 2^X, consisting of the family of nonempty closed subsets of X; $\mathcal{C}_n(X)$, whose elements belong to 2^X and each has

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