http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS

Volume 65, 2025

Pages 99–116

Local geometry of equilibria and a Poincaré-Bendixson-type theorem for holomorphic flows

by

Nicolas Kainz and Dirk Lebiedz

Electronically published on November 7, 2024

Topology Proceedings

Web:	http://topology.nipissingu.ca/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS
Volume 65 (2025)
Pages 99-116

E-Published on November 7, 2024

LOCAL GEOMETRY OF EQUILIBRIA AND A POINCARÉ-BENDIXSON-TYPE THEOREM FOR HOLOMORPHIC FLOWS

NICOLAS KAINZ AND DIRK LEBIEDZ

Abstract. In this paper, we explore the local geometry of dynamical systems $\dot{x} = F(x)$ with real time parameterization, where F is holomorphic on connected open subsets of $\mathbb{C} \cong \mathbb{R}^2$. We describe the geometry of first-order equilibria. For equilibria of higher orders, we establish an equivalent condition for "definite directions", allowing us to reverse the implication in Theorem 2 of Chapter 2.10 in [*Differential equations and dynamical systems*, Lawrence Perko (1990)] under the additional condition of holomorphy. This enables the geometric construction of a finite elliptic decomposition. We derive a holomorphic Poincaré-Bendixson-type theorem, leading to the conclusion that bounded non-periodic orbits are always homoclinic or heteroclinic.

1. Introduction

The qualitative topological description of the phase space of differential equations is an important area of research, cf. [1, 6, 15, 16], resulting in many significant insights into the local geometry of equilibria, as discussed in works like [2, 3]. Several of these results, especially regarding Newton flows, cf. [8, 12, 14, 17], are relevant for studying holomorphic and meromorphic flows of vectors fields derived from complex functions pertinent to

2020 *Mathematics Subject Classification.* Primary 30A99, 30C10, 30C15, 30D30; Secondary 32M25, 37F10, 37F75.
Key words and phrases. Holomorphic dynamical system, local geometry of equilibria, definite directions, finite elliptic decomposition, elliptic sectors, Poincaré-Bendixson.
Acknowledgement: We thank the referee for helpful suggestions and corrections.