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by

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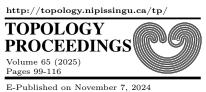
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LOCAL GEOMETRY OF EQUILIBRIA AND A POINCARÉ-BENDIXSON-TYPE THEOREM FOR HOLOMORPHIC FLOWS

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ABSTRACT. In this paper, we explore the local geometry of dynamical systems $\dot{x} = F(x)$ with real time parameterization, where F is holomorphic on connected open subsets of $\mathbb{C} \stackrel{\sim}{=} \mathbb{R}^2.$ We describe the geometry of first-order equilibria. For equilibria of higher orders, we establish an equivalent condition for "definite directions", allowing us to reverse the implication in Theorem 2 of Chapter 2.10 in [Differential equations and dynamical systems, Lawrence Perko (1990)] under the additional condition of holomorphy. This enables the geometric construction of a finite elliptic decomposition. We derive a holomorphic Poincaré-Bendixson-type theorem, leading to the conclusion that bounded non-periodic orbits are always homoclinic or heteroclinic.

1. INTRODUCTION

The qualitative topological description of the phase space of differential equations is an important area of research, cf. [1, 6, 15, 16], resulting in many significant insights into the local geometry of equilibria, as discussed in works like [2, 3]. Several of these results, especially regarding Newton flows, cf. [8, 12, 14, 17], are relevant for studying holomorphic and meromorphic flows of vectors fields derived from complex functions pertinent to

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