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# TOPOLOGY PROCEEDINGS



Volume 65, 2025

Pages 149–154

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## SMALL UNCOUNTABLE POWERS OF THE SORGENFREY LINE MAY NOT BE WEAKLY PSEUDOCOMPACT

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Electronically published on December 16, 2024

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**ISSN:** (Online) 2331-1290, (Print) 0146-4124

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## SMALL UNCOUNTABLE POWERS OF THE SORGENFREY LINE MAY NOT BE WEAKLY PSEUDOCOMPACT

REYNALDO ROJAS-HERNÁNDEZ

**ABSTRACT.** It is established that if Martin's Axiom  $MA(\kappa)$  holds for some cardinal  $\kappa$ , then the  $\kappa$ -power  $\mathbb{S}^\kappa$ , of the Sorgenfrey line  $\mathbb{S}$ , is not weakly pseudocompact.

### 1. INTRODUCTION

All spaces are assumed to be Tychonoff. A subspace  $X$  of  $Y$  is  $G_\delta$ -dense in  $Y$  if each nonempty  $G_\delta$ -set in  $Y$  contains a point of  $X$ . A well known result of Hewitt [6] states that a space  $X$  is pseudocompact if and only if  $X$  is  $G_\delta$ -dense in  $\beta X$ . Weak pseudocompactness was introduced by S. García-Ferreira and A. García-Máynez in [5]; a space  $X$  is *weakly pseudocompact* if it is  $G_\delta$ -dense in at least one of its compactifications. The interest in studying weakly pseudocompact spaces stems from the fact that this class of spaces contains all pseudocompact spaces and that all spaces in this class are Baire. A complete overview about weakly pseudocompact spaces can be found in [2].

While neither pseudocompactness nor the Baire property are finitely productive, weak pseudocompactness is a productive property. F. W. Eckertson proved in [3] that no countable power of the Sorgenfrey line is weakly pseudocompact and asked whether an uncountable power of the Sorgenfrey line is weakly pseudocompact; the case of the  $\omega_1$ -power of the

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2020 *Mathematics Subject Classification.* 54B10, 54D35, 54E52.

*Key words and phrases.* Sorgenfrey, pseudocompact, product spaces.

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