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ORDER TOPOLOGIES, AND CONTINUITY OF HOMOMORPHISMS

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Abstract. We consider ordered additive groups (rings, fields), and continuity of homomorphisms between them, in terms of their (algebraic) order topologies. Also, we give related examples.

1. Introduction

The symbol $\mathbb{Z}$ is the ring of integers with the usual order. Also, $\mathbb{Q}$ (resp. $\mathbb{R}$) is the field of rational numbers (resp. real numbers) with the usual order. $\mathbb{N}$ is the set of positive integers.

The topological terminologies used in this paper are elementary and well-known. For their definitions, see [2, 8], or [11], etc.

Let $X = (X, \leq)$ be a (linearly) ordered set having at least two points. For $a, b \in X$ with $a \leq b$, define the intervals $[a, b]$, (a, b) in X as in $\mathbb{R}$, and $(a, +\infty) = \{x \in X \mid a < x\}$, $(-\infty, a) = \{x \in X \mid x < a\}$.

$X = (X, \leq)$ is a *linearly ordered (topological) space* ([2], etc.), abbreviated *LOTS* ([8], etc.), if X has a *subbase* $\{(a, +\infty), (-\infty, a) \mid a \in X\}$. The topology on X is called the *order topology* (or *interval topology* ([3])), and we denote it by $\mathcal{T}(\leq)$. Every LOTS is *Hausdorff* (or, *normal*, as is well-known).

The symbol $(X, \leq)$ means a LOTS with the order topology $\mathcal{T}(\leq)$.

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Key words and phrases. order topology, continuous map, homomorphism, order-preserving map, connected space, ordered group, ordered ring, Archimedean property.

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