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TOPOLOGICAL ENTROPY OF LINEAR OPERATORS

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Abstract. We discuss the topological entropy of linear operators, specifically examining how the entropy is related to the operator's spectrum and to its norm. With a specific focus on linear operators on infinite dimensional Banach spaces, we show how the eigenvalues, the spectral radius, and the operator norm can be used to construct lower and upper bounds for the topological entropy.

1. Introduction

In this article, we examine the topological entropy of linear operators and the relationship between an operator's entropy and its spectrum. Topological entropy was introduced by R. L. Adler, A. G. Konheim, and M. H. McAndrew in [1]. We use an equivalent definition introduced by Rufus Bowen in [3]. For linear operators on finite dimensional vector spaces, a precise formula for the topological entropy in terms of the eigenvalues is known (see Theorem 2.5). For operators on infinite dimensional spaces, less is known, though there has been some research on the topic in recent years ([4], [5], [9]).

For linear operators on infinite dimensional Banach spaces, we examine the relationship between the spectrum and the topological entropy. Specifically, we show in section 3 that the eigenvalues of an operator can be used to compute a lower bound for the topological entropy, regardless of the dimension of the space, and, in section 4, we demonstrate relationships between an operator's entropy and its spectral radius and operator norm. We briefly discuss a connection to distributional chaos in

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