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ON STRONGER DYNAMICAL NOTIONS FOR A GENERAL NON-AUTONOMOUS DYNAMICAL SYSTEM

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Abstract. In this paper, we investigate the dynamics of a non-autonomous dynamical system $(I, \mathbb{F})$ generated by a sequence (f_n) of surjective continuous self maps on compact interval I converging uniformly to a surjective self map f. We prove that if a non-autonomous system on an interval is generated by a uniformly convergent sequence, then the system $(I, \mathbb{F})$ exhibits various notions of mixing (sensitivity) if the limiting system (I, f) exhibits the same. More generally, we prove that if the minimal radius for a ball that can be drawn inside $\omega_n(U)$ can be guaranteed, then the non-autonomous system $(X, \mathbb{F})$ exhibits stronger forms of mixing (sensitivities) if the limiting system (X, f) exhibits the same (and hence the implication holds in the absence of fast convergence). Consequently, we prove that strong dynamical behavior on an interval cannot arise on the boundary of plain dynamical systems (which do not exhibit stronger dynamical behavior).

Introduction

Let (X, d) be a compact metric space and let $\mathbb{F} = \{f_n : n \in \mathbb{N}\}$ be a family of continuous surjective self maps on X. For any given initial state of the system x_0, any such family generates a non-autonomous dynamical system via the relation $x_n = f_n(x_{n-1})$. Throughout the paper, we denote any such dynamical system as $(X, \mathbb{F})$. For any $x \in X$, $O_{\mathbb{F}}(x) = \{f_n \circ f_{n-1} \circ \ldots \circ f_1(x) : n \in \mathbb{N}\}$ defines the orbit of x. For notational convenience, let $\omega_n(x) = f_n \circ f_{n-1} \circ \ldots \circ f_1(x)$ (the state of the system after n iterations).

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