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EQUIVALENCES IN VARIATIONS OF SELECTION PRINCIPLES WITH PARTICULAR CLASSES OF FAMILIES OF SUBSETS OF A TOPOLOGICAL SPACE X

JUAN F. CAMASCA FERNÁNDEZ

Abstract. We present some results about the equivalences between the different variations in selection principles in certain types of classes of subsets of topological spaces, and we revisit particular cases.

1. Introduction

Let (X,τ) be a topological space and let $\mathcal{U}$ be an open cover of X.

Definition 1.1. We say that $\mathcal{U}$ is

- a ω-cover if $X \notin \mathcal{U}$ and for all finite subset $F \subset X$, there is $U \in \mathcal{U}$ such that $F \subset U$;
- a k-cover if $X \notin \mathcal{U}$ and for all compact subset $K \subseteq X$, there is $U \in \mathcal{U}$ such that $K \subseteq U$;
- a γ-cover if $X \notin \mathcal{U}$, $\mathcal{U}$ is infinite and for all $x \in X$, the set $\{U \in \mathcal{U} : x \notin U\}$ is finite;
- a large cover if for all $x \in X$ the set $\{U \in \mathcal{U} : x \in U\}$ is infinite.

Definition 1.2. Let (X,τ) be a topological space. A family $\mathbf{D}$ is said to be dense if all its elements are open sets and $\bigcup \mathbf{D}$ is dense in X.

Definition 1.3. Let (X,τ) be a topological space. A family $\mathbf{A}$ is cellular if all its elements are pairwise disjoint open sets. A family $\mathbf{A}$ is maximal

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