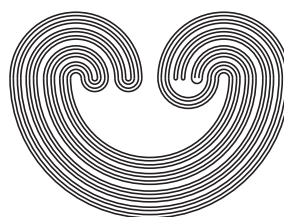


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## AN APPROXIMATION TOPOLOGY USING COUNTABLE ELEMENTARY SUBMODELS

by

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## AN APPROXIMATION TOPOLOGY USING COUNTABLE ELEMENTARY SUBMODELS

MAKOTO KUROSAKI

**ABSTRACT.** We introduce an approximation topology using countable elementary submodels and prove some classic theorems on generalized metric spaces.

### 1. INTRODUCTION

Since A. Dow's paper [3] was published, elementary submodels have been a standard tool in general topology. The aim of this paper is to give a method of using elementary submodels to show that a space is metrizable, paracompact, normal, etc. We introduce an approximation topology  $\mathcal{T}_a$  using countable elementary submodels in Lemma 2.4. An approximation topology  $\mathcal{T}_a$  for a space  $X$  is a coarser pseudometric topology and is grasped by countable elementary submodels  $\{M_x : x \in X\}$ . By using it, we give another proofs for some metrization theorems and theorems on normality of  $\Sigma$ -products in §3 and §4, respectively.

**Notation.** The symbol  $\omega$  denotes the first infinite ordinal. For a set  $A$ ,  $[A]^{<\omega}$  denotes the family of all finite subsets of  $A$ . For a space  $X$ ,  $\text{Top}(X)$  denotes the topology of  $X$ . For an  $x \in X$  and a cover  $\mathcal{U}$  of  $X$ ,  $\text{st}(x, \mathcal{U})$  denotes the set  $\bigcup\{U \in \mathcal{U} : x \in U\}$ . For families  $\mathcal{A}$  and  $\mathcal{B}$  of  $X$ ,  $\mathcal{A} \wedge \mathcal{B}$  denotes the family  $\{A \cap B : A \in \mathcal{A}, B \in \mathcal{B}\}$ .

In §2, 4.1, and 4.2, a space means a topological space without any separation axiom and in others,  $T_1$  and regular.

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