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by

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AN APPROXIMATION TOPOLOGY USING COUNTABLE ELEMENTARY SUBMODELS

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Abstract. We introduce an approximation topology using countable elementary submodels and prove some classic theorems on generalized metric spaces.

1. Introduction

Since A. Dow's paper [3] was published, elementary submodels have been a standard tool in general topology. The aim of this paper is to give a method of using elementary submodels to show that a space is metrizable, paracompact, normal, etc. We introduce an approximation topology $\mathcal{T}_a$ using countable elementary submodels in Lemma 2.4. An approximation topology $\mathcal{T}_a$ for a space X is a coarser pseudometric topology and is grasped by countable elementary submodels $\{M_x : x \in X\}$. By using it, we give another proofs for some metrization theorems and theorems on normality of Σ-products in §3 and §4, respectively.

Notation. The symbol ω denotes the first infinite ordinal. For a set A, $[A]^{<\omega}$ denotes the family of all finite subsets of A. For a space X, $\mathrm{Top}(X)$ denotes the topology of X. For an $x \in X$ and a cover $\mathcal{U}$ of X, $\mathrm{st}(x,\mathcal{U})$ denotes the set $\bigcup\{U \in \mathcal{U} : x \in U\}$. For families $\mathcal{A}$ and $\mathcal{B}$ of X, $\mathcal{A} \wedge \mathcal{B}$ denotes the family $\{A \cap B : A \in \mathcal{A}, B \in \mathcal{B}\}$.

In §2, 4.1, and 4.2, a space means a topological space without any separation axiom and in others, T_1 and regular.

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