http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS

Volume 66, 2025

Pages 201–218

Simple expansion sets and non-positive curvature

by

Daniel S. Farley

Electronically published on August 14, 2025

Topology Proceedings

Web:	http://topology.nipissingu.ca/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.nipissingu.ca/tp/
TOPOLOGY PROCEEDINGS
Volume 66 (2025)
Pages 201-218

E-Published on August 14, 2025

SIMPLE EXPANSION SETS AND NON-POSITIVE CURVATURE

DANIEL S. FARLEY

Abstract. An *expansion set* is a set $\mathcal{B}$ such that each $b \in \mathcal{B}$ is equipped with a set of expansions $\mathcal{E}(b)$. The theory of expansion sets offers a systematic approach to the construction of classifying spaces for generalized Thompson groups, as described in [6].

We say that $\mathcal{B}$ is *simple* if proper expansions are unique when they exist, or, equivalently, if $|\mathcal{E}(b)| \leqslant 2$ for all $b \in \mathcal{B}$.

We will prove that any given simple expansion set determines a cubical complex with a metric of non-positive curvature. In many cases, the cubical complex will be CAT(0). We are thus able to recover proofs that Thompson's groups F, T, and V [7,8], Houghton's groups H_n, and groups defined by finite similarity structures [9,11,14] all act on CAT(0) cubical complexes with finite stabilizers. We further state a sufficient condition for the cubical complex to be locally finite, and show that the latter condition is satisfied in the cases of F, T, V, and H_n.

1. Introduction

The class of generalized Thompson groups has been widely studied in recent years. Many such groups are characterized by local (or piecewise) definitions, such as Thompson's group V [5], the Brin-Thompson groups nV [2], Stein-Thompson groups [19], and Nekrashevych-Röver groups [16, 17], among others. In this introduction, we will use the term "generalized Thompson group" in a somewhat restricted sense, to refer to groups with such piecewise definitions. Our meaning of the term is thus broad enough to include the examples listed above, but, for instance, excludes the braided Thompson groups of [3].

2020 *Mathematics Subject Classification.* Primary 20F65; Secondary 20F67.
Key words and phrases. generalized Thompson groups, CAT(0) cubical complexes.