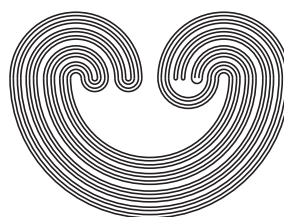


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## SIMPLE EXPANSION SETS AND NON-POSITIVE CURVATURE

by

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## SIMPLE EXPANSION SETS AND NON-POSITIVE CURVATURE

DANIEL S. FARLEY

**ABSTRACT.** An *expansion set* is a set  $\mathcal{B}$  such that each  $b \in \mathcal{B}$  is equipped with a set of expansions  $\mathcal{E}(b)$ . The theory of expansion sets offers a systematic approach to the construction of classifying spaces for generalized Thompson groups, as described in [6].

We say that  $\mathcal{B}$  is *simple* if proper expansions are unique when they exist, or, equivalently, if  $|\mathcal{E}(b)| \leq 2$  for all  $b \in \mathcal{B}$ .

We will prove that any given simple expansion set determines a cubical complex with a metric of non-positive curvature. In many cases, the cubical complex will be  $\text{CAT}(0)$ . We are thus able to recover proofs that Thompson's groups  $F$ ,  $T$ , and  $V$  [7, 8], Houghton's groups  $H_n$ , and groups defined by finite similarity structures [9, 11, 14] all act on  $\text{CAT}(0)$  cubical complexes with finite stabilizers. We further state a sufficient condition for the cubical complex to be locally finite, and show that the latter condition is satisfied in the cases of  $F$ ,  $T$ ,  $V$ , and  $H_n$ .

### 1. INTRODUCTION

The class of generalized Thompson groups has been widely studied in recent years. Many such groups are characterized by local (or piecewise) definitions, such as Thompson's group  $V$  [5], the Brin-Thompson groups  $nV$  [2], Stein-Thompson groups [19], and Nekrashevych-Röver groups [16, 17], among others. In this introduction, we will use the term “generalized Thompson group” in a somewhat restricted sense, to refer to groups with such piecewise definitions. Our meaning of the term is thus broad enough to include the examples listed above, but, for instance, excludes the braided Thompson groups of [3].

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