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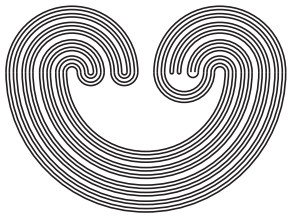

C-embedding, Lindelöfness, and Čech-completeness

by

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C-EMBEDDING, LINDELÖFNESS, AND ČECH-COMPLETENESS

ALAN DOW, KLAAS PIETER HART, JAN VAN MILL, AND HANS VERMEER

In memory of Gary Gruenhage

Abstract. We show that in the class of Lindelöf Čech-complete spaces the property of being C-embedded is quite well-behaved. It admits a useful characterization that can be used to show that products and perfect preimages of C-embedded spaces are again C-embedded. We also show that both properties, Lindelöf and Čech-complete, are needed in the product result.

Introduction

In [2] we investigated whether in realcompact spaces there could be closed, countable, and discrete subspaces (closed copies of the space $\mathbb{N}$ of natural numbers) that were C^*-embedded but not C-embedded, or even not C^*-embedded. In the follow-up paper [3] we looked for the smallest power of the real line $\mathbb{R}$ that could contain such closed copies of $\mathbb{N}$.

In the present paper we consider more general spaces. It appears that the members of the class of Lindelöf Čech-complete spaces behave much like $\mathbb{N}$ as regards C-embedding. Our positive results characterize C-embedding and allow us to conclude that, in this class, C-embedding is preserved by products and perfect preimages. We also show, by means of examples, that neither assumption, Lindelöfness nor Čech-completeness, can be dropped in these results.

2020 *Mathematics Subject Classification.* Primary 54C45; Secondary: 54D20, 54D35, 54D40, 54D60, 54G20.

Key words and phrases. C-embedding, Lindelöf, Čech-complete, product, perfect preimage.

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