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TOPOLOGY PROCEEDINGS

Volume 67, 2026

Pages 13–20

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Electronically published on May 1, 2025

Topology Proceedings

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E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.nipissingu.ca/tp/

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E-Published on May 1, 2025

RUDIN-TYPE DOWKER SPACES

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Abstract. A construction scheme of topological spaces, which generalizes M. E. Rudin's construction of a Dowker space in ZFC, is given, and is shown to produce a proper class of Dowker spaces. A proper subclass of this class of spaces are provably collectionwise normal Dowker in ZFC alone. The theory ZFC + SSH, where SSH is Shelah's Strong Hypothesis, proves that the whole class consists of collectionwise normal Dowker spaces. Whether all members of this class are Dowker in ZFC is still open.

1. Introduction

Rudin's construction of a Dowker space in ZFC [10] is one of the most elegant constructions in set-theoretic topology. Rudin's space, X^R, is a subset of the product $\prod_{0<n}(\aleph_n + 1)$, with the topology inherited from the box topology on the full product. Its topological properties were established by Rudin via, somewhat surprisingly, typical PCF theory arguments — about twenty years before Shelah developed PCF theory.

Recently, A. Rinot asked if there was a proper class of Dowker spaces in ZFC alone, and, more specifically, whether replacing the sequence of $\aleph_n$-s in Rudin's construction by the sequence of all finite successors, μ^{+n} for some positive natural number n, of an arbitrary cardinal μ, can be proved in ZFC to always yields a Dowker space.

2020 *Mathematics Subject Classification.* 54A35, 54G20, 03E04.

Key words and phrases. Dowker spaces, Rudiń́s space, pcf theory, Shelah´s Strong Hypohesis.