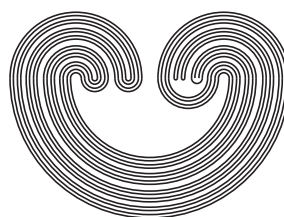


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RUDIN-TYPE DOWKER SPACES

by

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RUDIN-TYPE DOWKER SPACES

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ABSTRACT. A construction scheme of topological spaces, which generalizes M. E. Rudin's construction of a Dowker space in \mathbf{ZFC} , is given, and is shown to produce a proper class of Dowker spaces. A proper subclass of this class of spaces are provably collectionwise normal Dowker in \mathbf{ZFC} alone. The theory $\mathbf{ZFC} + \mathbf{SSH}$, where \mathbf{SSH} is Shelah's Strong Hypothesis, proves that the whole class consists of collectionwise normal Dowker spaces. Whether all members of this class are Dowker in \mathbf{ZFC} is still open.

1. INTRODUCTION

Rudin's construction of a Dowker space in \mathbf{ZFC} [10] is one of the most elegant constructions in set-theoretic topology. Rudin's space, X^R , is a subset of the product $\prod_{0 < n} (\aleph_n + 1)$, with the topology inherited from the box topology on the full product. Its topological properties were established by Rudin via, somewhat surprisingly, typical PCF theory arguments — about twenty years before Shelah developed PCF theory.

Recently, A. Rinot asked if there was a proper class of Dowker spaces in \mathbf{ZFC} alone, and, more specifically, whether replacing the sequence of \aleph_n -s in Rudin's construction by the sequence of all finite successors, μ^{+n} for some positive natural number n , of an arbitrary cardinal μ , can be proved in \mathbf{ZFC} to always yields a Dowker space.

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