

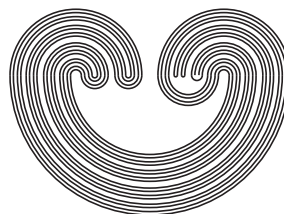
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## A NOTE ON PREIMAGES OF COUNTABLE-TO-ONE CONTINUOUS MAPS

by

ARKADY LEIDERMAN AND PAUL SZEPTYCKI

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## A NOTE ON PREIMAGES OF COUNTABLE-TO-ONE CONTINUOUS MAPS

ARKADY LEIDERMAN  AND PAUL SZEPTYCKI 

*To the memory of Gary Gruenhage, a great man and topologist*

**ABSTRACT.** Let  $\mathcal{C}$  be a class of topological spaces and suppose that  $f : X \rightarrow Y$  is a continuous countable-to-one surjective map, does  $Y \in \mathcal{C}$  imply that  $X \in \mathcal{C}$ ?

We consider this question for various classes  $\mathcal{C}$  including the class of  $\Delta_1$ -spaces and its subclasses. One of the main results obtained in this paper says that if  $X$  is a compact space and  $Y$  is a scattered Eberlein compact space then so is  $X$ .

We examine various consistent examples of continuous 2-to-1 surjective maps  $f : X \rightarrow Y$  such that  $Y$  is a  $Q$ -set of reals, but  $X$  is not a  $Q$ -space. Several open questions which we pose in the paper are surprisingly related to the old and still unsettled problem asking whether there is a  $\Delta$ -set of reals which is not a  $Q$ -set.

### 1. INTRODUCTION

Throughout the paper we consider only Tychonoff topological spaces. A mapping  $f : X \rightarrow Y$  is called *countable-to-one* (*finite-to-one*) if all fibers  $f^{-1}(y), y \in Y$  are at most countable (finite, respectively).

**Question 1.** *Let  $\mathcal{C}$  be a class of topological spaces and suppose that  $f : X \rightarrow Y$  is a continuous countable-to-one surjective mapping, does  $Y \in \mathcal{C}$  imply that  $X \in \mathcal{C}$ ?*

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*Key words and phrases.* countable-to-one continuous map,  $Q$ -set,  $\Delta$ -set, Eberlein compact, effectively  $\Delta$ -space.

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