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A NOTE ON PREIMAGES OF COUNTABLE-TO-ONE CONTINUOUS MAPS

ARKADY LEIDERMAN AND PAUL SZEPTYCKI

To the memory of Gary Gruenhage, a great man and topologist

Abstract. Let $\mathcal{C}$ be a class of topological spaces and suppose that $f : X \to Y$ is a continuous countable-to-one surjective map, does $Y \in \mathcal{C}$ imply that $X \in \mathcal{C}$?

We consider this question for various classes $\mathcal{C}$ including the class of Δ_1-spaces and its subclasses. One of the main results obtained in this paper says that if X is a compact space and Y is a scattered Eberlein compact space then so is X.

We examine various consistent examples of continuous 2-to-1 surjective maps $f : X \to Y$ such that Y is a Q-set of reals, but X is not a Q-space. Several open questions which we pose in the paper are surprisingly related to the old and still unsettled problem asking whether there is a Δ-set of reals which is not a Q-set.

1. Introduction

Throughout the paper we consider only Tychonoff topological spaces. A mapping $f : X \to Y$ is called *countable-to-one* (*finite-to-one*) if all fibers $f^{-1}(y), y \in Y$ are at most countable (finite, respectively).

Question 1. *Let $\mathcal{C}$ be a class of topological spaces and suppose that $f : X \to Y$ is a continuous countable-to-one surjective mapping, does $Y \in \mathcal{C}$ imply that $X \in \mathcal{C}$?*

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Key words and phrases. countable-to-one continuous map, Q-set, Δ-set, Eberlein compact, effectively Δ-space.

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