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ORDERABILITY OF SPACES HAVING ORDERED DECOMPOSITIONS

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Abstract. The following may be well-known:

- the subspace $(0,1) \cup \{2\}$ of the usual real numbers $\mathbb{R}$ is the topological sum of two linearly ordered spaces, and well-known that there is no linear ordering of X whose open interval topology coincides with the topology of X.

In this paper, we consider when the topological sum of a pairwise disjoint collection $\mathcal{X}$ of ordered spaces are orderable. As corollaries, we see:

- whenever $\mathcal{X}$ contains infinitely many singletons or contains an infinite discrete space, its topological sum is orderable;
- whenever $\mathcal{X}$ contains at least one ordered space with a maximal element but without minimal elements, its topological sum is orderable;
- whenever $\mathcal{X}$ does not contain ordered spaces with both a maximal element and a minimal element, its topological sum is orderable;
- whenever $\mathcal{X}$ contains infinitely many ordered spaces with both a maximal element and a minimal element, its topological sum is orderable;
- whenever $\mathcal{X}$ consists of suborderable spaces, its topological sum is suborderable.

Let $<$ be a linear order on a set X, see [3, page 4]. The pair $\langle X, < \rangle$ is said to be a linearly ordered set or an ordered set, and usually simply

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