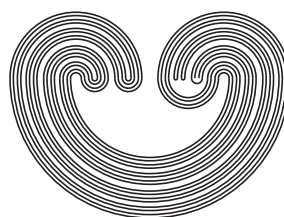

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ORDERABILITY OF SPACES HAVING ORDERED DECOMPOSITIONS

by

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ORDERABILITY OF SPACES HAVING ORDERED DECOMPOSITIONS

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ABSTRACT. The following may be well-known:

- the subspace $(0, 1) \cup \{2\}$ of the usual real numbers \mathbb{R} is the topological sum of two linearly ordered spaces, and well-known that there is no linear ordering of X whose open interval topology coincides with the topology of X .

In this paper, we consider when the topological sum of a pairwise disjoint collection \mathcal{X} of ordered spaces are orderable. As corollaries, we see:

- whenever \mathcal{X} contains infinitely many singletons or contains an infinite discrete space, its topological sum is orderable;
- whenever \mathcal{X} contains at least one ordered space with a maximal element but without minimal elements, its topological sum is orderable;
- whenever \mathcal{X} does not contain ordered spaces with both a maximal element and a minimal element, its topological sum is orderable;
- whenever \mathcal{X} contains infinitely many ordered spaces with both a maximal element and a minimal element, its topological sum is orderable;
- whenever \mathcal{X} consists of suborderable spaces, its topological sum is suborderable.

Let $<$ be a linear order on a set X , see [3, page 4]. The pair $\langle X, < \rangle$ is said to be a linearly ordered set or an ordered set, and usually simply

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