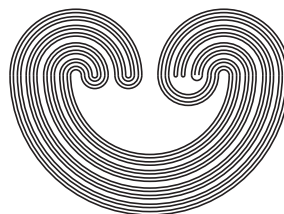


TOPOLOGY PROCEEDINGS



Volume 67, 2026

Pages 81–94

INVERSE LIMITS OF HAUSDORFF ARCS

by

MICHEL SMITH

Electronically published on June 30, 2025

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.nipissingu.ca/tp/subscriptioninfo.html> for information.

Topology Proceedings

Web: <http://topology.nipissingu.ca/tp/>

Mail: Topology Proceedings
Department of Mathematics & Statistics
Auburn University, Alabama 36849, USA

E-mail: topolog@auburn.edu

ISSN: (Online) 2331-1290, (Print) 0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.

INVERSE LIMITS OF HAUSDORFF ARCS

MICHEL SMITH

ABSTRACT. We prove that if $\mathbf{X}$ is an inverse limit of Hausdorff arcs and $\mathbf{X}$ is a hereditarily indecomposable continuum, then each coordinate arc has countable cellularity (satisfies the countable chain condition). From this theorem and previous results it follows that if $\mathbf{X}$ is an inverse limit of Hausdorff arcs and $M \subset \mathbf{X}$ is a hereditarily indecomposable continuum, then M is a metric continuum.

1. INTRODUCTION

By a *Hausdorff arc* is meant a nondegenerate Hausdorff continuum with exactly two non-cut points; equivalently, a *Hausdorff arc* is a compact connected LOTS (linearly ordered topological space.) A topological space is said to satisfy the countable chain condition (ccc) if there is no uncountable collection of disjoint open sets in the space.

The Souslin Problem asks: if a LOTS has neither a first nor last element, is dense in itself, complete and satisfies the ccc condition then is it homeomorphic to the reals? [19] A connected LOTS that satisfies the countable chain condition and is not homeomorphic to the reals is called a Souslin line and a compact non-degenerate connected subset of a Souslin line is called a Souslin arc. The Souslin Hypothesis is that there is no Souslin line. It has been shown that the existence of a Souslin line, and therefore the Souslin Hypothesis, is independent of ZFC (see Jech [7] and Solovay and Tennenbaum [18]).

2000 *Mathematics Subject Classification.* Primary 54F15, 54D35; Secondary 54B20.

Key words and phrases. continua, indecomposable continua, hereditarily indecomposable continua, Hausdorff spaces, Souslin Hypothesis.

©2025 Topology Proceedings.

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.nipissingu.ca/tp/subscriptioninfo.html> for information.