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INVERSE LIMITS OF HAUSDORFF ARCS

MICHEL SMITH

Abstract. We prove that if $\mathbf{X}$ is an inverse limit of Hausdorff arcs and $\mathbf{X}$ is a hereditarily indecomposable continuum, then each coordinate arc has countable cellularity (satisfies the countable chain condition). From this theorem and previous results it follows that if $\mathbf{X}$ is an inverse limit of Hausdorff arcs and $M \subset \mathbf{X}$ is a hereditarily indecomposable continuum, then M is a metric continuum.

1. Introduction

By a *Hausdorff arc* is meant a nondegenerate Hausdorff continuum with exactly two non-cut points; equivalently, a *Hausdorff arc* is a compact connected LOTS (linearly ordered topological space.) A topological space is said to satisfy the countable chain condition (ccc) if there in no uncountable collection of disjoint open sets in the space.

The Souslin Problem asks: if a LOTS has neither a first nor last element, is dense in itself, complete and satisfies the ccc condition then is it homeomorphic to the reals? [19] A connected LOTS that satisfies the countable chain condition and is not homeomorphic to the reals is called a Souslin line and a compact non-degenerate connected subset of a Souslin line is called a Souslin arc. The Souslin Hypothesis is that there is no Souslin line. It has been shown that the existence of a Souslin line, and therefore the Souslin Hypothesis, is independent of ZFC (see Jech [7] and Solovay and Tennenbaum [18]).

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