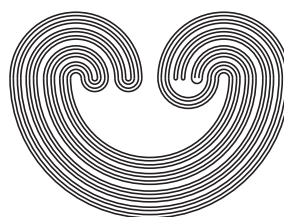


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by

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GRUENHAGE GAMES ON GROUPS

JOCELYN BELL AND STEVEN CLONTZ

ABSTRACT. One of Gary Gruenhage's earliest contributions to topology appeared in the form of a topological game called the W -game. His W -game has been played ever since, and in this paper we explore his original game and the proximal game, a game that was inspired by the W -game, in the context of topological groups.

1. INTRODUCTION

In his dissertation and a subsequent paper published in 1976 [7], Gary Gruenhage introduced what he called a “simple two-person infinite game” defined on a topological space. Since then, his game has proven to be quite useful for characterizing certain topological properties, and has inspired several variations [9]. Such topological games typically involve two players alternately choosing elements or subsets of a topological space. If one of the players can win no matter what the other does, then that player is said to have a winning strategy. The existence of winning strategies in topological games often implies certain topological properties hold [17]. In this paper we examine Gruenhage's game and a related yet different game in the context of topological groups, leading to our main result:

Main Result (Corollary 5). *Countably compact W -groups are normal.*

Throughout this paper we assume topological spaces are Hausdorff.

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