http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS

Volume 67, 2026

Pages 95–110

Gruenhage Games on Groups

by

Jocelyn Bell and Steven Clontz

Electronically published on July 28, 2025

Topology Proceedings

Web:	http://topology.nipissingu.ca/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.nipissingu.ca/tp/
TOPOLOGY PROCEEDINGS
Volume 67 (2026)
Pages 95-110
E-Published on July 28, 2025

GRUENHAGE GAMES ON GROUPS

JOCELYN BELL AND STEVEN CLONTZ

Abstract. One of Gary Gruenhage's earliest contributions to topology appeared in the form of a topological game called the W-game. His W-game has been played ever since, and in this paper we explore his original game and the proximal game, a game that was inspired by the W-game, in the context of topological groups.

1. Introduction

In his dissertation and a subsequent paper published in 1976 [7], Gary Gruenhage introduced what he called a "simple two-person infinite game" defined on a topological space. Since then, his game has proven to be quite useful for characterizing certain topological properties, and has inspired several variations [9]. Such topological games typically involve two players alternately choosing elements or subsets of a topological space. If one of the players can win no matter what the other does, then that player is said to have a winning strategy. The existence of winning strategies in topological games often implies certain topological properties hold [17]. In this paper we examine Gruenhage's game and a related yet different game in the context of topological groups, leading to our main result:

Main Result (Corollary 5)**.** *Countably compact W-groups are normal.*

Throughout this paper we assume topological spaces are Hausdorff.

2020 *Mathematics Subject Classification.* 91A44, 54E15, 22A05.
Key words and phrases. W-space, topological group, proximal game, proximal space.