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TOPOLOGY PROCEEDINGS

Volume 67, 2026

Pages 125–141

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Yasushi Hirata and Yukinobu Yajima

Electronically published on July 28, 2025

Topology Proceedings

Web:	http://topology.nipissingu.ca/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.nipissingu.ca/tp/
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Volume 67 (2026)
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E-Published on July 28, 2025

IRREDUCIBILITY ON MONOTONICALLY NORMAL SPACES AND GO-SPACES

YASUSHI HIRATA AND YUKINOBU YAJIMA

Abstract. We first present a characterization for the irreducibility of monotonically normal spaces. Making use of it, we proceed another characterization for the irreducibility of GO-spaces. Next, we discuss when GO-spaces have an irreducible subspace which is not D. As a result, we give a characterization for GO-spaces which have a closed irreducible subspace being not D.

1. Introduction

The study of D-spaces has been advanced by many mathematicians since around 2000. The concept was first introduced by van Douwen in [8]. However, a substantial start seems to be the theorem of van Douwen and Lutzer [7] in 1997. Combining a classical result of Engelking and Lutzer [9], it is stated as follows.

Theorem 1.1. *For a GO-space X, the following are equivalent.*

(a) *X is paracompact.*
(b) *X is weakly submetalindelöf.*
(c) *There is no closed subspace in X homeomorphic to a stationary subset in a regular uncountable cardinal.*
(d) *X is a D-space.*

2020 *Mathematics Subject Classification.* 54A25, 54D15, 54D20, 54F05.
Key words and phrases. Irreducible, monotonically normal, GO-space, D-space, extent, Lindelöf degree.
This research was supported by Grant-in-Aid for Scientific Research (C) 23K03206.