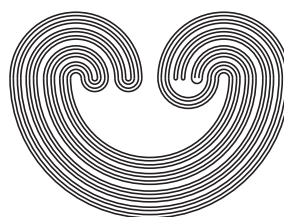


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## GRUENHAGE SPACES AND THEIR INFLUENCE ON BANACH SPACE RENORMING THEORY

by

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## GRUENHAGE SPACES AND THEIR INFLUENCE ON BANACH SPACE RENORMING THEORY

RICHARD J. SMITH

*This article is dedicated to the memory of Prof. Gary F. Gruenhage (1947 – 2023).*

**ABSTRACT.** In a paper from 1987, Gruenhage defined a class of topological spaces that now bear his name, and used it to solve a problem of Talagrand on the existence of dense  $G_\delta$  metrizable subsets of Gul'ko compact spaces. Gruenhage's paper became highly influential among researchers in renorming theory, a branch of Banach space theory. In this paper we survey Gruenhage and related spaces, and their interactions with renorming theory.

### 1. INTRODUCTION

Let  $X$  be a topological space. In this paper we shall consider only Hausdorff spaces. We call a family  $\mathcal{U}$  of subsets of  $X$   $T_0$ -*separating* or simply *separating* if, given distinct  $x, y \in X$ , there exists  $U \in \mathcal{U}$  such that  $\{x, y\} \cap U$  is a singleton (so in general it is not possible to choose  $U$  so that  $\{x, y\} \cap U = \{x\}$ ). Given a family  $\mathcal{U}$  of subsets of  $X$  and  $x \in X$ , we write

$$\text{ord}(x, \mathcal{U}) = \text{card} \{U \in \mathcal{U} : x \in U\}.$$

In his brief yet influential 1987 study of Gul'ko compact spaces, Gruenhage defined the following concept.

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