

PSEUDO-ISOMETRIC SURGERY

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ABSTRACT. We introduce a type of surgery on metric spaces. This surgery, in some sense, seeks to replace a subspace S of a metric space X with another metric space T via a function $f\colon S\to T$. When T is a discrete space, this amounts to collapsing the subspace according to the function. This surgery results in a new metric space we denote \widehat{X}_f and there is a natural function $F\colon X\to \widehat{X}_f$ induced from f. Our primary interest is investigating if properties of the original function f are inherited by the induced function F. We show that if f is a pseudo-isometry then so is F. However, for a quasi-isometry, a very natural generalization of a pseudo-isometry that is prevalent in geometric group theory, such a result does not hold.

1. Introduction

The idea of removing a subset from a space and replacing it with a modified version is one the most basic transformations of mathematics. For example the Möbius band, often obtained as the result of a cut/twist/reglue operation can also arise from a remove/alter/replace operation on the annulus, see Figure 1. Such transformations are used to produce new spaces that are simulatneously different from, yet similar to, the original.

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