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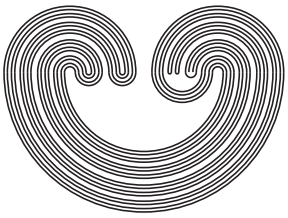

A pair of monads in Topology

by

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A PAIR OF MONADS IN TOPOLOGY

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Abstract. Two monads of interest arise from the dual adjunction between the category of topological spaces and that of (bounded) distributive lattices. These are the open prime filter monad and the ideal lattice monad. It is known that the ideal lattice monad induces the ideal frame comonad on the category of frames. We show that this ideal frame comonad can be paired with the open prime filter monad via the open set-spectrum adjunction. From this, we give a new proof of the equivalence between the category of stably compact spaces and that of stably compact frames on one hand, and that of compact Hausdorff spaces and compact regular frames on the other. We show, among other things, how the Čech-Stone compactifications in Pointfree Topology and Pointset Topology relate to each other in this particular context.

1. Introduction

What could be considered as the main results of this paper at first glance are not new, and to a certain extent may be seen as a folklore by some authors. By that, we refer to the equivalence between the category **StKSp** of stably compact spaces and the category **StKFrm** of stably compact frames[1] on one hand, and that between the category **KHaus** of compact Hausdorff spaces and the category **KRegFrm** of compact regular frames on the other. The paper attempts to understand these

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Key words and phrases. monad, Eilenberg-Moore algebras, stably compact spaces, frames and locales, coherent frames, compactifications, prime filters, ideals, distributive lattices.

[1]The terminology *Stably continuous frames* is also used in the literature (Cf. [8]).