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THE TOPOLOGY OF A TOPOLOGICAL SUM OF ORDERABLE SPACES IS INDUCED BY THE UNION OF TWO ORDER TOPOLOGIES

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Abstract. It is known that the subspace $X = (0,1) \cup \{2\}$ in the real line $\mathbb{R}$ is the topological sum of the ordered subspaces $(0,1)$ and $\{2\}$ of $\mathbb{R}$ which is not orderable. In this paper, we prove that the topology of a topological sum of orderable spaces is induced by the union of two order topologies.

In the present paper [4], the orderability of the topological sum of orderable spaces was discussed. Among other results, the orderability of the topological sum of locally connected orderable spaces was characterized, as a corollary, the subspace $(0,1) \cup \{2\}$ in the real line $\mathbb{R}$ is a locally connected suborderable space which is not orderable. The following classical results are known:

- for every subordered space $\langle X, <, \tau \rangle$, there are $\max\{|M|, 2\}$-many orders on X such that the union of the corresponding order topologies induce the original topology τ, where M denotes the set of missing points in it, see [6],
- the topology of the Sorgenfrey line is induced by the union of two order topologies on it, see [5].

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