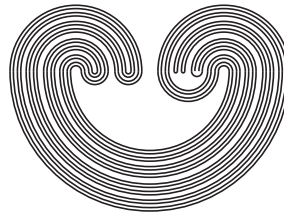


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THE TOPOLOGY OF A TOPOLOGICAL SUM OF
ORDERABLE SPACES IS INDUCED BY THE UNION
OF TWO ORDER TOPOLOGIES

by

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**THE TOPOLOGY OF A TOPOLOGICAL SUM OF
ORDERABLE SPACES IS INDUCED BY THE UNION OF
TWO ORDER TOPOLOGIES**

NOBUYUKI KEMOTO

ABSTRACT. It is known that the subspace $X = (0, 1) \cup \{2\}$ in the real line \mathbb{R} is the topological sum of the ordered subspaces $(0, 1)$ and $\{2\}$ of \mathbb{R} which is not orderable. In this paper, we prove that the topology of a topological sum of orderable spaces is induced by the union of two order topologies.

In the present paper [4], the orderability of the topological sum of orderable spaces was discussed. Among other results, the orderability of the topological sum of locally connected orderable spaces was characterized, as a corollary, the subspace $(0, 1) \cup \{2\}$ in the real line \mathbb{R} is a locally connected suborderable space which is not orderable. The following classical results are known:

- for every subordered space $\langle X, <, \tau \rangle$, there are $\max\{|M|, 2\}$ -many orders on X such that the union of the corresponding order topologies induce the original topology τ , where M denotes the set of missing points in it, see [6],
- the topology of the Sorgenfrey line is induced by the union of two order topologies on it, see [5].

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