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by

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GEOMETRY OF CANONICAL REGIONS AND ELLIPTIC SECTORS IN HOLOMORPHIC FLOWS

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Abstract. In this follow-up paper, we investigate the global geometry and topology of dynamical systems $\dot{x} = F(x)$ with entire vector field F, building on and constructively extending the local structure of simple and higher-order equilibria. We provide a step-by-step analysis to reveal topological properties of the basins of centers, nodes, and foci, while excluding isolated equilibria at the boundaries of the latter two. We propose a definition of global elliptic sectors and introduce the concept of sector-forming orbits based on the geometry within a finite elliptic decomposition of multiple equilibria. Finally, we characterize the structure of heteroclinic regions connecting two equilibria.

1. Introduction

In our previous work [18], we examined the local structure of simple and higher-order equilibria in holomorphic vector fields, i.e. in *holomorphic dynamical systems* of the form

$$\dot{x} = \frac{\mathrm{d}x}{\mathrm{d}t} = F(x), \quad x \in \mathbb{C},\ t \in \mathbb{R} \tag{1.1}$$

with $F \in \mathcal{O}(\mathbb{C})$. Building on these local insights, this follow-up paper shifts the perspective to the global phase portrait of entire vector fields and develops an understanding of how local structures influence and shape

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Key words and phrases. Holomorphic dynamical system, entire vector field, center, period annulus, focus, node, basin of attraction/stability, elliptic sector, canonical region.

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