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TOPOLOGY PROCEEDINGS

Volume 68, 2026

Pages 123–135

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Electronically published on January 22, 2026

Topology Proceedings

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E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.nipissingu.ca/tp/

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E-Published on January 22, 2026

PERIODIC POINTS, INDUCED HYPERSPACE MAPS AND LI-YORKE CHAOS

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Abstract. Let X be a continuum, and let $f : X \to X$ be a homeomorphism. Let 2^X be the hyperspace of all nonempty compact subsets of X endowed with the Hausdorff metric. Let $C(X) \subset 2^X$ be the collection of all subcontinua of X. Let $2^f : 2^X \to 2^X$ be the induced map defined by $2^f(A) = f(A)$, and let $C(f)$ be the restriction of 2^f to $C(X)$. Let $Per(f)$ and $R(f)$ be the set of all periodic points of f and the set of all recurrent points of f respectively. In this note we prove the following: If $f : X \to X$ is a dendrite homeomorphism with $R(f) = X$, then the induced homeomorphism 2^f has no Li-Yorke pairs. In the second part we produce a dendroid homeomorphism $f : X \to X$ such that $Per(f) = X$, and the induced map $C(f)$ is chaotic in the sense of Li-Yorke, that is, there exists an uncountable subset S of $C(X)$, such that for each pair $A, B \in S$, with $A \neq B$, (A, B) is a Li-Yorke pair of $C(f)$.

1. Introduction and some definitions

Let X be a compact metric space, and let $f : X \to X$ be a continuous function. The pair (X, f) is a discrete dynamical system. Let $\mathbb{N}$ denote the set of all positive integers. For each $n \in \mathbb{N}$, let $f^n : X \to X$ denote the n-th *composition* of f. A point $x \in X$ is said to be:

(i) a *fixed point* of f if $f(x) = x$;

2020 *Mathematics Subject Classification.* Primary 37B05, 54B20, 54F16; Secondary 37B20.

Key words and phrases. Periodic points, Li-Yorke pairs, induced hyperspace maps.