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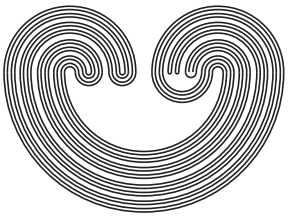

Proper maps and quasi-adjoints

by

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PROPER MAPS AND QUASI-ADJOINTS

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Abstract. We show that a continuous map is proper if only if it has a quasi-adjoint, namely a left adjoint in the Kleisli category of the Smyth hyperspace monad. As applications, we show that the Smyth and Hoare hyperspace functors, as well as the continuous valuation functor, preserve proper maps. We also show that any projective limit of consonant sober spaces with proper bonding maps is consonant and sober. No separation axiom is assumed.

1. Introduction

The main object of study of this paper is *proper maps*. We will show that those are characterized as the continuous maps that have what we call a *quasi-adjoint* (Section 3), a natural construction that arises from the consideration of the Kleisli category of the Smyth hyperspace monad $\mathcal{Q}_0$ (Section 4). We will apply this in Section 5 and show that the Smyth and Hoare hyperspace functors, as well as the continuous, subprobability and probability valuation functors, preserve proper maps; and in Section 6, where we will show that projective limits of ($\odot$-)consonant sober spaces with proper bonding maps are ($\odot$-)consonant and sober.

2. Preliminaries

We consider topological spaces that are not necessarily Hausdorff or T_1, and we refer to [9]. Every topological space X has a *specialization*

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