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by

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ON CONTINUOUS POLYNOMIALS OF THE MACÍAS SPACE

JHIXON MACÍAS

To my wife and our first child

ABSTRACT. Let $\mathbb{N}$ be the set of natural numbers. The Macías space $M(\mathbb{N})$ is the topological space $(\mathbb{N}, \tau_M)$ where τ_M is generated by the collection of sets $\sigma_n := \{m \in \mathbb{N} : \gcd(n, m) = 1\}$. In this paper, we characterize the continuity of polynomial and exponential functions over $M(\mathbb{N})$; and prove that the only continuous polynomials are monomials, and that the only continuous exponential functions are of the form $f(x) = a^x$.

1. INTRODUCTION

In 1953, M. Brown introduced a topology τ_G on the set of natural numbers $\mathbb{N}$, which is generated by the collection of arithmetic progressions $b(\mathbb{N} \cup \{0\}) + a$ where $a, b \in \mathbb{N}$ such that $\gcd(a, b) = 1$ (here the symbol $\gcd(a, b)$ means the greatest common divisor of a and b). In [14, Counterexample 60] the topology τ_G is called the *relatively prime integer topology.* It was not until 1959 that the topology introduced by Brown was popularized by S. Golomb (now known as Golomb's topology) who in [7] proves that Dirichlet's theorem on arithmetic progressions is equivalent to the set of prime numbers being dense in the topological space $(\mathbb{N}, \tau_G)$.

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