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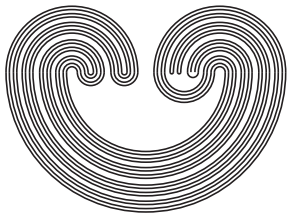

Groups acting on products of locally finite trees

by

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GROUPS ACTING ON PRODUCTS OF LOCALLY FINITE TREES

J. O. BUTTON

Abstract. We examine the question of which finitely generated groups act properly on a finite product of locally finite simplicial trees and present evidence in favour of hyperbolic surface groups having such an action. We also give a completely explicit embedding of the genus 2 closed hyperbolic surface group in $SL_2(\mathbb{F}_p(x, y))$ for any prime p.

1. Introduction

A useful technique which can be used to understand a finitely generated group G is to find a geodesic metric space X on which it acts geometrically, that is properly and cocompactly by isometries. Of course every G has such an action on its Cayley graph but if the space X is well behaved geometrically, for instance if it is a hyperbolic space or a CAT(0) space, then this allows us to establish properties for the group G.

However it can be quite restrictive asking for the action to be geometric. Indeed a group G admits a geometric action on some simply connected geodesic metric space if and only if G is finitely presented ([2] I.8.11), so if G has a geometric action on such a space but G also has a subgroup H which is finitely generated but not finitely presented then there is no geometric action of H on any simply connected geodesic metric space.

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