Laminations of the Unit Disk and Julia Sets

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A complex polynomial P of degree≥ 2 always has a nonempty, perfect, nowhere dense fully invariant subset $J(P)$ of the complex plane called its *Julia set*. On this set the map P is *chaotic*. By way of a classical theorem on change of variables around a fixed critical point, the Böttkher Uniformization Theorem, a connection can be made between the topological and dynamical structure of (connected) Julia sets and invariant laminations of the unit disk, potentially a friendlier object of study. For example, if we parameterize the unit circle by $[0, 1)$, the map on the unit circle which corresponds to a cubic polynomial is $t \mapsto 3t \pmod 1$, a (deceptively) very simple map, until one considers what its invariant subsets on the circle are. In this talk, we define invariant laminations of the unit disk and their relation, in particular, to quadratic and cubic Julia sets. We use this as a jumping-off point for open questions about Julia sets and invariant laminations, and recent progress made on some of them.