13th Annual Workshop on Topology and Dynamical Systems
Nipissing University, May 16–20, 2016
http://topology.nipissingu.ca/workshop2016/

Dynamical properties of shift maps of inverse limits with a set valued function

Judy Kennedy (Lamar University)
judy.kennedy@lamar.edu

Joint with: Van Nall (University of Richmond)
vnall@richmond.edu

Abstract: Set valued functions from an interval into the closed subsets of an interval arise in various areas of science and mathematical modeling. Research has shown that the dynamics of a single valued function on a compact space are closely linked to the dynamics of the shift map on the inverse limit with the function as the sole bonding map. For example, it has been shown that with Devaney's definition of chaos the bonding function is chaotic if and only if the shift map is chaotic. One reason for caring about this connection is that the shift map is a homeomorphism on the inverse limit, and therefore the topological structure of the inverse limit space must reflect in its richness the dynamics of the shift map. In the set valued case there may not be a natural definition for chaos since there is not a single well defined orbit for each point. However, the shift map is a continuous single valued function so it together with the inverse limit space form a dynamical system which can be chaotic in any of the usual senses. For the set valued case we demonstrate with theorems and examples rich topological structure in the inverse limit when the shift map is chaotic (on certain invariant sets). We then connect that chaos to a property of the set valued function that is a natural generalization of an important chaos producing property of continuous functions.