15th Annual Workshop on Topology and Dynamical Systems
Nipissing University, May 21–25, 2018
http://topology.nipissingu.ca/workshop2018/

Colocally connected, non-cut, non-block, and shore sets in symmetric products

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Abstract: Given a continuum X, we consider its hyperspaces:

- $2^X = \{A \subset X : A \text{ is closed and nonempty}\}$,
- $C_n(X) = \{A \in 2^X : A \text{ has at most } n \text{ components}\}$,
- $F_n(X) = \{A \in 2^X : A \text{ has at most } n \text{ points}\}$.

and these hyperspaces are endowed with the Hausdorff metric H.

Definition. Let X be a continuum and A a subcontinuum of X with $\operatorname{int}(A) = \emptyset$. We say that A is

(1) a continuum of colocal connectedness in X provided that for each open subset U of X with $A \subset U$ there exists an open subset V of X such that $A \subset V \subset U$ and $X \setminus V$ is connected.

(2) not a weak cut continuum in X if for any pair of points $x, y \in X$ with $x, y \notin A$ there is a subcontinuum M of X such that $x, y \in M$ and $M \cap A = \emptyset$.

(3) a nonblock continuum in X provided that there exist a sequence of subcontinua $M_1, M_2, \ldots$ of X such that $M_1 \subset M_2 \subset \cdots$ and $\bigcup M_n$ is a dense subset of $X \setminus A$.

(4) a shore continuum in X if for each $\varepsilon > 0$ there is a subcontinuum M of X such that $H(M, X) < \varepsilon$ and $M \cap A = \emptyset$.

(5) not a strong center in X provided that for each pair of nonempty open subsets U and V of X there exists a subcontinuum M of X such that $M \cap U \neq \emptyset \neq M \cap V$ and $M \cap A = \emptyset$.

(6) a noncut continuum in X if $X \setminus A$ is connected.

It is an easy exercise to prove that any property with smaller number implies one with a bigger number. J. Bobok et al. made a detailed study of properties (1)–(6) when A is a singleton, in particular, they gave examples to show that none of the reverse implications are true (see pages 240–241).

We prove that for every continuum X and every $n \in \mathbb{N}$, $F_1(X)$ is a nonblock continuum in $F_n(X)$. However this is not always true for conditions (1) & (2).

For condition (1) we prove that if X is locally connected then for every $n \in \mathbb{N}$, $F_1(X)$ is a continuum of colocal connectedness in $F_n(X)$.

For condition (2) we prove that if X is an arcwise connected continuum then for every $n \in \mathbb{N}$, $F_1(X)$ is not a weak cut continuum in $F_n(X)$.

We prove that for every continuum X and every $n \geq 3$, $F_1(X)$ is a continuum of colocal connectedness in $F_n(X)$.

We show that for the Knaster continuum, $F_1(X)$ is not a continuum of colocal connectedness in $F_2(X)$, but even further we prove that an arcwise connected continuum as simple as harmonic fan does not satisfy that $F_1(X)$ is a continuum of colocal connectedness in $F_n(X)$. In general $F_m(X)$ is a continuum of colocal connectedness in $F_n(X)$ with $m \leq n-2$. Finally we prove that for $\sin(1/x)$ curve $F_1(X)$ is a weak cut continuum in $F_n(X)$.