

Topological and uniform structures on universal covering spaces

Nikolay Brodskiy (University of Tennessee, Knoxville)
nbrodskiy@gmail.com

Abstract: We discuss three topologies on the universal covering space (the set of fixed endpoint homotopy classes of based paths in a topological space): the quotient topology, the whisker topology introduced by Spanier, and the lasso topology introduced by the authors in an earlier paper. Using the concept of universal Peano space we describe a similarity between the quotient topology and the whisker topology. We introduce the concept of small loop transfer space to characterize when the different topologies coincide.

We also discuss three uniform structures on the universal covering space of a uniform space: the James uniform structure (analogous to the quotient topology), the Berestovskii-Plaut uniform structure (analogous to the whisker topology), and the lasso uniform structure. We show that while the James uniformity might not exist, the lasso uniformity exists for any uniform space and coincides with James uniformity for path connected and uniformly locally path connected spaces. We use the concept of uniform small loop transfer space to characterize when the lasso uniformity and the Berestovskii-Plaut uniformity coincide.