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Mutiplicity of Hexagon Numbers

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Abstract: Imagine you have an unlimited supply of congruent equilateral triangles. *Polygon numbers* are the number of these triangles used to tile a convex polygon. *Triangle numbers* are n^2 where n is the side length of a tiled equilateral triangle. We can use the method of removing equilateral triangles from a large tiled equilateral triangle to create equiangular hexagons, as well as certain parallelograms, trapezoids, and pentagons. Thus, $H = n^2 - (a^2 + b^2 + c^2)$ is a *hexagon number*, where a, b, and c are the side lengths of the equilateral triangles removed from the corners.

It is known that all natural numbers $H \geq 85$ are hexagon numbers (OEIS A229757). The *multiplicity* of a polygon number P is the number of ways to construct P up to congruence. A polygon number P with multiplicity 1 is said to be *unique*. Previously, two of the authors had shown (1) there are infinately many unique trapezoid numbers, and (2) for every natural number n, there exists a hexagon number H_n that has multiplicity $m(H_n) \geq n$. One of our current results is the theorem: there exists a natural number $N \leq 288$ such that every hexagon number $H \geq N$ has multiplicity $m(H) \geq 2$. That is, there are only finitely many unique hexagon numbers. Numerical explorations suggest to us the following conjecture: for every natural number m, there is a natural number G_m, such that for every hexagon number $H \geq G_m$, $m(H) \geq m$. Further numerical exploration has lead us to investigate currently the role played by Pythagorean primes (primes $p \equiv 1 \pmod 4$) on the multiplicity of hexagon numbers.